Probability Distributions - I

Binomial Distribution

Distributions are broadly classified into two headings
1) Observed frequency Distribution
2) Theoretical or Probability Distribution.

Observed frequency Distribution: The distributions which are obtained by actual observations or experiments are called observed frequency Distribution. for Example

Marks	0-10	10-20	20-30	30-40
No of students	5	15	20	25

2. Theoretical or Probability Distribution: The distributions which are not obtained by actual observations or experiments but are mathematically deduced under certain assumptions are called Theoretical distributions. These are also called Probability distribution or Expected frequency distributions.

Types of Theoretical or Probability Distributions

- Discrete P.D
  - Binomial Distribution
  - Poisson Distribution
- Continuous P.D
  - Normal Distribution

Main Parameters (n and p)

Constant -

Binomial Distribution

It is a discrete P.D. It is used when an experiments [r]esults in conditions - success (P) and failure (q)

[B]inomial Distribution is defined by

$$P(X=x) = {}^{n}C_{x}\, q^{n-x} \cdot p^{x} \quad (x = 0, 1, 2, \ldots)$$

where P = Probability of success

q = Probability of failure = 1 - P

n = number of trials

P(X=x) = Probability of x successes in n trials.

Conditions or Assumptions.

Finite number of trials

Mutually Exclusive outcomes

Probability of success (P) in Each trial is constant.

Trials are independent

Properties or Characterstics

1. Theoretical frequency Distribution
2. Discrete Probability Distribution.
3. Line Graph ()
4. Shape of Binomial Distribution (depends on value of P, q)
   If $P = \frac{1}{2}$, then symmetrical
   If $P \neq \frac{1}{2}$, then asymmetrical

5. Main Parameters (n and p)

6. constants

Mean $\bar{x} = np$

variance $\sigma^2 = npq$

S.D $= \sigma = \sqrt{npq}$

Moment coeff. of skewness

$$\sqrt{\beta_1} = \frac{q-p}{\sqrt{npq}}$$

moment coeff. of kurtosis

$$\beta_2 = 3 + \frac{1-6pq}{npq}$$

7. uses (success and failure)

Probability Distributions - II
Poisson Distribution

Poisson Distribution is a limiting case of binomial distribution under conditions

I n, the number of trials is very large i.e $n \to \infty$

II p (Prob. of success) is very small, q (Prob. of failure) is very large $\}\to p \to 0,\ q \to 1$

III The average number of successes (np) = +ve finite quantity (m) i.e $np = m$

Definition : Poisson distribution is defined by

$$P(X=x) = e^{-m} \cdot \frac{m^x}{x!} \quad \text{for } x = 0, 1, 2, \ldots$$

where $P(X=x)$ = Prob of x number of success

$m = np$ = Parameter

$e = 2.7183$

operties

Discrete Prob. distribution

value of P, q (it is used when prob of occurance is very small i.e $P \to 0$
& Prob. of non occurance is very large $(q \to 1)$
(& n is very large)

Main Parameter (m, $m = nP$)

shape of P.D. (always positively skewed)

Constants

$\bar{X} = m = nP$ moment coeff of skewness $\sqrt{\beta_1} = \frac{1}{\sqrt{m}}$

variance $= \sigma^2 = m$

$S.D = \sigma = \sqrt{m}$ moment coeff of kurtosis $\beta_2 = 3 + \frac{1}{m}$

VI Equality of mean & variance $(\bar{X} = \sigma^2)$

Role / uses / Importance of Poisson Distribution

1. To count number of defects of an item (statistical)
2. To count number of bacteria (in Biology)
3. To count number of casualities (in Insurance)
4. To count number of typing errors per page in typed material

5. To count no. of incoming telephone calls in town
6. " " " " detective blades in a lot of blades
7. " " " " deaths in a town as result of accidents (factory)

" " suicides committed in a year.

Probability Distributions - III

Normal Distribution

Normal Dist. is one of the most important & widely used cont. prob. dist.

Normal Distribution is limiting form of Binomial Distribution under conditions

1. n is infinitely large $n \to \infty$
2. Neither P nor q is small

Def^n : $P(X=x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{X-\bar{X}}{\sigma}\right)^2}$, $-\infty < X < +\infty$

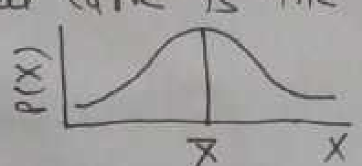
In standard normal variate form

$$P(Z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2} \quad \left(Z = \frac{X-\bar{X}}{\sigma} \right)$$

in Standard Normal dist. $\rightarrow$ mean of $Z = 0$, $\sigma = 1$

Graph of Normal Distribution : Its Graph is called normal curve

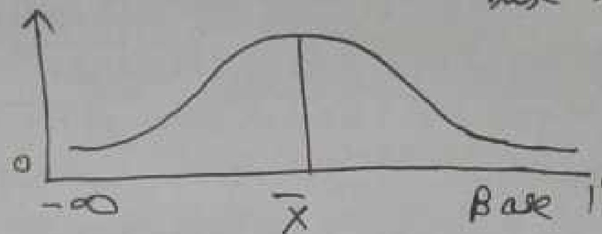
i.e. Normal curve is the graphical Presentation of Normal Dist.



Characteristics / Properties

1. Perfectly symmetrical and Bell shaped
2. Unimodal Distribution (only one mode)
3. equality of Mean, Median, Mode ($\bar{X} = M = Z$)

4. Asymptotic to Base line | it has tendency to touch the base line but it never touches it)



(6)

(5) Range $(-\infty$ to $+\infty)$
6 Total area (unity = 1)
7. ordinate (height (ordinate) of normal curve is maximum)
8. Mean ordinate (two parts 50% on right & 50% on left side)

9. Equidistance of Quartiles $(Q_3 - M = M - Q_1)$
10. Quartile Deviation $(Q.D = \frac{2}{3} S.D)$

11 Mean Deviation $(M.D = \frac{4}{5} S.D)$

12. Points of Inflexion
13. Continuous Prob. Dist.
14. constants : $\bar{x}$ or μ or m

$SD = \sigma$ Mom. coef of Skewness $= \sqrt{\beta_1} = 0$
$var. = \sigma^2$ " " " Kurtosis $\beta_2 = 3$

15. Parameters : $\bar{X}$, σ
16. Areas Property.

Importance : (1) study of Natural Phenomenon
2. Basis of Sampling Theory
3 Statistical quality control
4. useful for large sample Test
5. Approximation to Binomial, Poisson Dist.

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* Relation b/w Binomial, Normal Distribution

Binomial $\rightarrow$ normal under condition

(i) $n \rightarrow \infty$

(ii) Neither p nor q is very small.

* Relation b/w Poisson and Normal

Poisson $\rightarrow$ normal if its parameter $m \rightarrow \infty$

* Diff. b/w Normal (N.D) and Binomial (B.D) Dist.

1. Nature $\rightarrow$ B.D is discrete Prob. Dist. but N.D is continuous P.D.

2. Probability f^n: f^n of B.D is
$$P(X=x) = {}^nC_x\, q^{n-x}\, p^x$$
f^n of N.D is
$$P(X=x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{X-\bar{X}}{\sigma}\right)^2}$$

3. Value of n: In B.D n is finite but in N.D $n \rightarrow \infty$

4. Values of p, q $\rightarrow$ B.D $p, q = 0.5$ (app.)
N.D Neither p nor q is very small

5. Parameters
In B.D Parameters are n, p
N.D — $\bar{X}, \sigma$.

6. Shape: In B.D symm. and asymm. (depends on p, q)
In N.D (Perfectly symm.)

4. Acumptotic to Base line | it has tendency to touch the

Diff. b/w Normal and Poisson Dist.

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	Normal	Poisson
1	Nature continuous	Discrete
2	$P(X=x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2}$	$P(X=x) = e^{-m} \cdot \frac{m^x}{x!}$
3	n is very large	n is very large
4.	Neither p nor q is very small	$p \to 0$, $q \to 1$
5.	Parameters $\bar{x}, \sigma$	m
6.	Shape Perfectly symmetrical	positively skewed in shape

* Comparison of Binomial, Poisson & Normal Dist.

	Properties	Binomial	Poisson	Normal
1.	Nature	Discrete	Discrete	Continuous
2.	Prob. function	$P = {}^nC_x q^{n-x} p^x$	$P = e^{-m} \frac{m^x}{x!}$	$P = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2}$
3	Parameter	n, p $0 < p < 1$	m	$\bar{x}, \sigma$
4.	Limiting form	—	$n \to \infty$ $p \to 0$ $np \to m$	$n \to \infty$ Neither p nor q is small
5.	Mean & Var.	$\bar{x} = np$ $\sigma^2 = npq$	$\bar{x} = m$ $\sigma^2 = m$	$\bar{x}$ or m σ^2
6	Shape	Symmetrical or Asymmetrical	Positively skewed	Perfectly Symmetrical