DISTRIBUTION

Binomial Distribution:→ is a discrete Probability distribution. This distribution was discovered by a Swiss Mathematician James Bernoulli. It is used in such situation where an Experiment result in two possibilities → Success or failure. Binomial Distribution is a discrete Prob. distribution which Expresses the Probability of one set of two alternatives Success (P) and failure (q)

Definition of Binomial Distribution:—

Binomial distribution is defined and given by the following probability density function:—

$P(x)\ {}^{n}C_{x}\, q^{n-x} . P^{x}$

Where:—

p = Probability of Success

q = Probability of failure $(1-P)$

n = No. of trials

x = No. of Successes in n trial

By Substituting the different values of X in the above Probability functions of the Binomial distribution we can obtain the Prob. of 0, 1, 2 — — n successes as follows

Number of Success (x)	Probability $P(x)$
0	${}^{n}C_{0}\, q^{n-0} p^{0} = q^{n}$
1	${}^{n}C_{1}\, q^{n-1} p^{1} = n q^{n-1} p^{1}$
2	${}^{n}C_{2}\, q^{n-2} p^{2} = \frac{n(n-1)}{2\times 1} q^{n-2} p^{2}$
⋮	⋮
n	${}^{n}C_{n}\, q^{n-n} p^{n} = p^{n}$

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Conditions or Assumption of Binomial Distribution:-

Binomial Distribution can be used only under the following Conditions:-

1 finite no. of trials:-

Under Bino. dist. an Experiment is Performed Under Identical Conditions for a finite and fixed no. of trials i.e. no. of trials are finite.

2 Mutually Exclusive Outcomes:-

Each trial must Result in two Mutually Exclusive outcomes success or failure. for exp. If a Coin is tossed then Either the Head or tail may be turn up.

3 The Probability of Success in each trial is Same:- In each trial the prob of of Success denoted by P remain Constant. In other words the Prob. of Success in different trials does not change. for example:- In tossing a Coin the prob. of getting a head in Each toss remains Same i.e. $P = P(H) = 1/2$

4 Trails are Independent:-

In Binomial distribution Statistical Independent among trails is Assumed i.e. the outcomes of any trial does not effect the outcomes of the subsequent trials.

Properties / Characterstics of Binomial Distributions

The following are the Important properties or characterstics of Binomial distribution:—

1 **Theoritical frequency distribution** :—

A Binomial Dist. is a theoretical freq. distribution which is based on binomial theorem of algebra with the help of this distribution. We can obtain the theoretical frequency by Multiplying the Probability of Success by the total number.

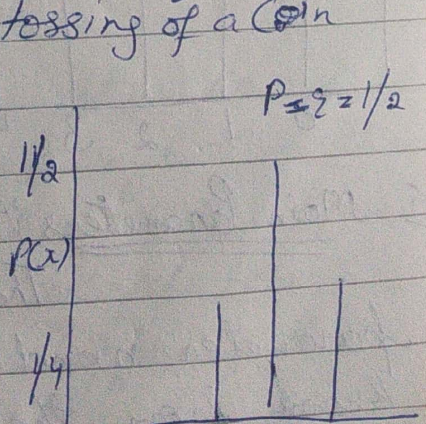
2 **Discrete Probability Distribution** :—

The binomial distribution is a discrete Prob. dist. in which the number of successes 0, 1, 2, 3 --- n are given in whole numbers and not in fraction.

3 **line graph** :—

The binomial dist. can be presented graphically by means of a line graph. The no. of success is taken on the X-axis and the Prob. of success taken on the Y-axis. The following line graph is based on the tossing of a coin twice.

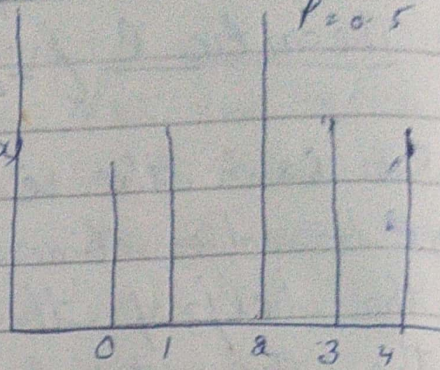
No. of head x	Prob. P_x
0	$2C_0\left(\frac{1}{2}\right)^2 = 1/4$
1	$2C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^1 = 1/2$
2	$2C_2\left(\frac{1}{2}\right)^2 = 1/4$



4 Shape of Binomial Distribution :-

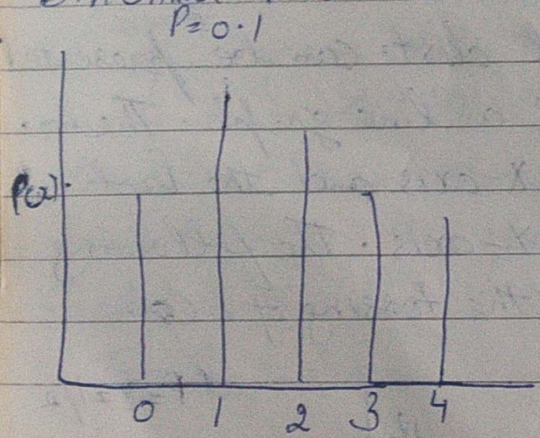
The Shape of Binomial dist. is depends on the value of P, q and n which is Shown Below

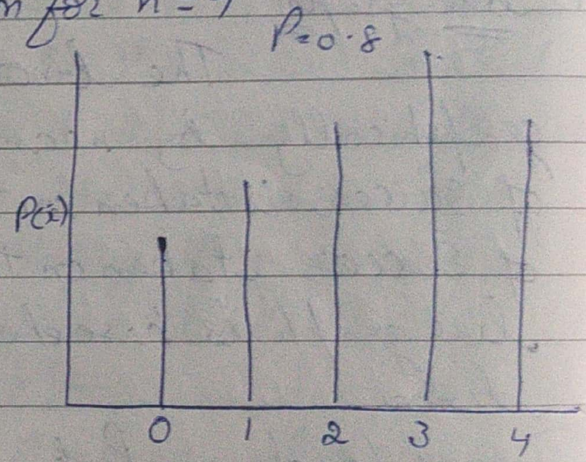
(i) If $P = q = 1/2$ the bin. dist is Perfectly Symmetrical for any value of n



(ii) If $P \neq q \neq 1/2$ the binomial dist. will be asymmetrical i.e. the binomial distribution is skewed. If $P < q$ the dist. will be +ve Skewed and If $P > q$ the dist. will be -ve Skewed. If the value of n Increases for $P \neq q$ the asymmetry or Skewness in the distr. diminishes.

Binomial Prob. Distribution for $n = 4$





5 Main Parameters :-

The binomial distribution has two parameter n and P. The entire distribution can be known from these two parameters.

6 Mean and Standard Deviation :-

The mean and Standard deviation of binomial distribution can be obtained by using the following formula:—

$$\text{mean} = (\bar{x}) = np$$

$$S.D = \sigma = \sqrt{npq}$$

$$\text{Variance} = \sigma^2 = npq$$

Ⅰ **Uses**:—

It has been found useful in those fields where the outcomes is classified in to success and failure. In other words it is useful in Coin experiment, dice throwing, Manufacturing of Item by a Company etc.

Imporance of Binomial Distribution :-

The Binomial Probability Distribution is a discrete Prob. dist. that is useful in descri-bing an Enormous variety of real life Events.

for Example :- a quality Control Inspector wants to know the Prob. of defective light bulbs in a random Sample of 10 bulbs if 10% of the bulbs are defective. He can quickly obtain the answer from tables of the Binomial Prob. distribution. The Bino-mial dist. can be used when :-

1. The Outcome or Result of each trial in the Process are characterised as one of two types of Possible outcomes. In other words they are attributes.
2. The Possibility of Outcome of any trial does not change and is Independent of the Result of Previous trials.

Example :-

A Coin is tossed Six times. What is the Prob. of obtaining four or More ~~heads~~ heads.

Conditional Probability

The multiplication theorem not applicable in case of dependent Events. Dependent Events are those in which the occurance of one Event affects the prob. of other Events. The Prob. of the Event B given that A has occurred is called the Conditional Probability of B. It is denoted by $P(B/A)$. Similarly the Conditional Prob. of A given that B has occured is denoted by $P(A/B)$

Definition of Conditional Probability :-

If the Events A and B are dependent, the Conditional Prob. of B given A is defined and Given by :-

$$P(B/A) = \frac{P(AB)}{P(A)} \quad \text{Where } P(A) > 0$$

Similarly the Conditional Prob. of A given B is defined ~~as~~ and given by :-

$$P(A/B) = \frac{P(AB)}{P(B)} \quad \text{where } P(B) > 0$$

$A \cap B \cap C$

MULTIPLICATION THEOREM

This Theorem States that If two events A and B are Independent, the Prob. that they Both will occur is equal to the Product of Their Individual

Prob. Symbolically:
If A and B are Independent, then
$P(A \text{ and } B) = P(A) \times P(B)$
The Theorem can be Extended to three or more Independent Events. Thus
$P(A, B \text{ and } C) = P(A) \times P(B) \times C$

Proof of the Theorem:-

If an Event A can happen in n_1 ways of which a_1 are Successful and the Event B can happen in n_2 ways of of which a_2 are Successful. We can Combine Each Successful Event in the Second Case. Thus the total number of Successful happening in both Cases is $a_1 \times a_2$. Similarly the total no. of Possible cases in $n_1 \times n_2$.
Then by definition the Prob. of the occurance of both Events is

$$\frac{a_1 \times a_2}{n_1 \times n_2} = \frac{a_1}{n_1} \times \frac{a_2}{n_2}$$

But $\frac{a_1}{n_1} = P(A)$ $\quad \frac{a_2}{n_2} = P(B)$

$\therefore P(A \text{ and } B) = P(A) \times P(B)$
In a Similar way the theorem can be Extended to three or More Events

OR

Multiplication Theorem for Independent Events:-

If A and B are two Independent Events then the Prob. of the Simultaneous occurance of A and B is the Multiplication of their Separate Probabilities of A and B.

~~Symb~~ Symbolically

$P(AB) = P(A \text{ and } B) = P(A) \times P(B)$

Proof:-

let m_1 Be the no. of favourable ~~c~~ to Event A

n_1 " " " " Equally likely cases to A

$\therefore$

$$P(A) = \frac{m_1}{n_1}$$

And:-

let m_2 be the no. of favourable to Event B and ~~ne~~ n_2 be the no. of Equally likely Cases to B.

$$\therefore \; P(B) = \frac{m_2}{n_2}$$

Now, by the fundamental Principal of ~~Count~~ Counting the no. of Cases favourable to the occurance of the Event AB is $m_1 m_2$ out of $n_1 n_2$

$$\therefore \; P(AB) = \frac{m_1 m_2}{n_1 n_2}$$

$$P(AB) = \left(\frac{m_1}{n_1}\right)\left(\frac{m_2}{n_2}\right)$$

$$P(AB) = P(A) \times P(B)$$

Hence Proved

Multiplication Theorem in case of dependent Event
OR
Multiplication theorem in case of Conditional Prob :-

If A and B are two dependent Events then the Prob of Simultaneous Occurance is the Prob of one multiplied by Conditional Prob of the other.

As we Know that :-

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A/B) \cdot P(B)$$

As like :-

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow P(B \cap A) = P(B/A) \cdot P(A)$$

Now :-
We will Solve this theorem by following Example :-
A Bag Contains 6 Red and 4 Black Ball. We draw two balls one after another without Replacement. find out the Prob that these ball will be Red and Black.

Solution :-
Prob. of getting Red ball from bag $P(A) = 6/10$
" " " " Black " " " " $P(B) = 4/10$

This Occurance Shows two Solution :-
Ist is bag Shows Red ball in Ist trial and Shows black ball in IInd Trial

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$$P(B \cap A) = P(B/A) \cdot P(A)$$
$$\frac{4}{9} \times \frac{6}{10} = \frac{24}{90}$$

IInd is :-

Bag give black ball in Ist trial and IInd ball in Second trial

Now :-

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

or

$$P(A \cap B) = P(A/B) \cdot P(B)$$
$$= \frac{6}{9} \times \frac{4}{10} = \frac{24}{90}$$

This Example Proves that :-

$$P(A \cap B) = P(B \cap A)$$
$$\frac{24}{90} = \frac{24}{90}$$

OR

$$P(A \cap B) = P(A/B) \cdot P(B) = P(B \cap A) = P(B/A) \cdot P(A)$$